Model Repairing And Belief Change Operators

Daniel Grimaldi[1,2,3,*,†], Edwin Pin Baque[4,†]

[1]University of Cape Town and CAIR, Private Bag X3, Rondebosch 7701, South Africa
[2]Departamento de Computación, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria, C1428EGA, Argentina
[3]Instituto de Investigación en Ciencias de la Computación, UBA-CONICET, Ciudad Universitaria, C1428EGA, Argentina
[4]Departamento de Matemática, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria, C1428EGA, Argentina

Abstract

This paper proposes a framework for database repairing that abstracts away from specific choices of language and structure, assuming only a primitive relation between a set of structures and a set of formulas. Within this framework, we define a family of belief change operators that capture an abstract notion of repair, characterized by partial orders and a set of admissible outcomes. This means that, given an initial structure and a set of constraints, the operator can impose extra-logical criteria on the class of admissible structures. In particular, this captures distance-based, subset, and superset repairing.

Keywords

Knowledge Reasoning, Logic, Belief Change Theory, Repairing

1. Introduction

Repairing addresses the problem of minimally modifying a structure so as to satisfy a given set of constraints. Classical approaches in database theory and consistent query answering provide a standardized account of repairs based on minimality criteria, such as distance or set inclusion [1, 2] (see Example 3.5), or on cardinality conditions [3], among others. In [4], the authors studied the problem of modeling repairs using belief change operators in the particular framework of inconsistencies over Computational Tree Logic and Kripke structures. Inspired by this idea, we propose a generalized approach built on the Abstract Worlds Semantics framework [5], which captures the semantics of AGM theory [6], and assumes a primitive relation between a set of structures and a language, inducing a notion of consequence. Within this setting, we define a family of belief change operators based on [7, 8]. These operators take a structure and a set of constraints and return admissible repairs determined by a partial order over structures and extra-logical restrictions on outcomes, such as subset or superset inclusions. This yields a uniform account of distance-based, subset, and superset repairing.

In summary, we propose a common language between model repair and belief change theory, beyond a specific language and its logic. This would facilitate the transfer of implementation techniques from model repair to belief change while providing model repair with a stronger theoretical foundation. Moreover, the framework admits an alternative reading in terms of Formal Concept Analysis [9], opening further connections between these areas.

The work is organized as follows. In Section 2, we define the general framework for languages and structures. In Section 3, we present the family of belief change operators that capture repairing, axiomatically and constructively, and present a representation theorem. We also add some concrete examples related to known languages. Lastly, in Section 4, we discuss future work. Due to limited space, proofs of the results are presented in the Appendix.

Logical Approaches to Handling Inconsistent Data (LINDA), July 24th, 2026, Lisbon, Portugal

*Corresponding author.
†These authors contributed equally.
dgrimaldi@dc.uba.ar (D. Grimaldi); epin@dc.uba.ar (E. Pin Baque)
0009-0000-6131-2372 (D. Grimaldi); 0000-0002-4783-3531 (E. Pin Baque)

2. The General Setup

In [5], the author defines a framework known as Abstract Worlds Semantics (AWS), which achieves separation of the semantic behavior of belief change theory from the logical structure typically associated with it, reconnecting them via an antitone Galois connection [10]. The framework is based on the algebra of an arbitrary index set, whose elements, called worlds, represent primitive constructions that can eventually be associated with specific structures. Our proposal builds on this framework by associating worlds with structures capable of interpreting formulas of a language.

Definition 2.1. *Given a language $\mathbb{L}$, a set of structures $\mathcal{S}$, and a relation $\models\ \subseteq \mathcal{S} \times \mathbb{L}$, written $s \models \psi$, we say they define a* **logic** $\mathcal{L} = \langle \mathbb{L}, \mathcal{S}, \models \rangle$. *A logic is always associated with the following functions:*

- $\mathbb{W} : \mathcal{P}(\mathbb{L}) \to \mathcal{P}(\mathcal{S})$ *such that* $\mathbb{W}(\Gamma) = \bigcap_{\psi \in \Gamma} W_\psi$, *where* $W_\psi = \{s \in \mathcal{S} \mid s \models \psi\}$.
- $\mathbb{T} : \mathcal{P}(\mathcal{S}) \to \mathcal{P}(\mathbb{L})$ *such that* $\mathbb{T}(A) = \bigcap_{s \in A} T_s$, *where* $T_s = \{\psi \in \mathbb{L} \mid s \models \psi\}$.
- $Cn = \mathbb{T} \circ \mathbb{W}$ *and* $Cl = \mathbb{W} \circ \mathbb{T}$.

The previous definition not only connects $\mathbb{L}$ and $\mathcal{S}$ to perceive them as a logic, but it also starts developing a connection with the AWS framework for belief change theory.

Theorem 2.2. *Let $\mathcal{L} = \langle \mathbb{L}, \mathcal{S}, \models \rangle$. Then the pair $(\mathbb{W}, \mathbb{T})$ is an antitone Galois connection, i.e.:*

$$A \subseteq \mathbb{W}(\Gamma) \qquad \text{if and only if} \qquad \Gamma \subseteq \mathbb{T}(A) \qquad \forall\, A \in \mathcal{P}(\mathcal{S})\,,\ \Gamma \in \mathcal{P}(\mathbb{L}) \tag{1}$$

Also, given $Y \subseteq \mathcal{P}(\mathcal{S})$ and $X \subseteq \mathcal{P}(\mathbb{L})$, and extending the functions as $\mathbb{T}(Y) = \{\mathbb{T}(A) \mid A \in Y\}$ and $\mathbb{W}(X) = \{\mathbb{W}(\Gamma) \mid \Gamma \in X\}$, the following properties are satisfied:

1. $\mathbb{W}$ *and* $\mathbb{T}$ *are order reversing (i.e., they invert the inclusion order);*
2. $Cn = \mathbb{T} \circ \mathbb{W}$ *and* $Cl = \mathbb{W} \circ \mathbb{T}$ *are Tarskian operators;*
3. $\mathbb{W}(\Gamma) = \mathbb{W}(Cn(\Gamma)) = Cl(\mathbb{W}(\Gamma))$ *and* $\mathbb{T}(A) = Cn(\mathbb{T}(A)) = \mathbb{T}(Cl(A))$;
4. $\mathbb{W}$ *and* $\mathbb{T}$ *define isomorphisms between* $(\mathcal{K} = \{K \subseteq \mathbb{L} \mid K = Cn(K)\}, \subseteq, \cap, \oplus, Cn(\emptyset), \mathbb{L})$ *the lattice of theories, where* $\bigoplus X = Cn(\bigcup X)$, *and* $(\mathcal{C} = \{A \subseteq \mathcal{S} \mid A = Cl(A)\}, \supseteq, \oplus, \cap, \mathcal{S}, Cl(\emptyset))$ *the lattice of closed sets, where* $\bigoplus Y = Cl(\bigcup Y)$.

The lattice isomorphism of Theorem 2.2 defines a translation between the set of structures and the language, based on the relation $\models$. In particular, it is possible to connect a set of formulas with the set of structures that validate them. This setup can also be understood under Formal Concept Analysis, where $(\mathcal{S}, \mathbb{L}, \models)$ is known as a formal context, and the pairs (A, K) where $K = \mathbb{T}(A)$ and $A = \mathbb{W}(K)$ are formal concepts. However, analyzing this perspective is beyond the scope of this work. Our objective is to present a family of belief change operators in this framework, as in the next section, that captures the repairing of a given structure.

3. Model Repairing as a Belief Change Operator

In this section, we adapt credibility-limited (CL) update operators [8] for $\mathcal{P}(\mathcal{S})$ based on the idea that (a) when dealing with repair, having one structure s in the first parameter, as in [7], connects with the local nature of update; (b) we look for the minimal structures, based on a partial order $\leq_s$, that satisfy a set of constraints Γ represented by formulas, i.e., the minimal elements of $\mathbb{W}(\Gamma)$ w.r.t $\leq_s$; (c) the credible set adds extra-logic constraints, such as in the context of subset or superset repairs.

Definition 3.1. *We say $\circ_\mathcal{S} : \mathcal{P}(\mathcal{S}) \times \mathcal{P}(\mathcal{S}) \to \mathcal{P}(\mathcal{S})$ is a credibility-limited (CL) $\mathcal{S}$-update if it satisfies the following postulates:*

(S∘1) $A \circ_\mathcal{S} B \subseteq B$ *(success)*

(S∘2) *If $A \subseteq B$ then $A \circ_{\mathcal{S}} B = A$ (weak vacuity)*

(S∘3) $A \circ_{\mathcal{S}} B = \bigcup_{s \in A} (\{s\} \circ_{\mathcal{S}} B)$ *(pointwise)*

(S∘4) *If $A \circ_{\mathcal{S}} B \neq \emptyset$ and $B \subseteq C$ are finite sets, then $A \circ_{\mathcal{S}} C \neq \emptyset$ (finite consistent propagation)*

(S∘5) $(A \circ_{\mathcal{S}} B) \cap C \subseteq A \circ_{\mathcal{S}} (B \cap C)$ *(superexpansion)*

(S∘6) *If $s \in \mathcal{S}$ and $\emptyset \neq \{s\} \circ_{\mathcal{S}} B \subseteq C \subseteq B$, then $\{s\} \circ_{\mathcal{S}} B = \{s\} \circ_{\mathcal{S}} C$ (weak pointwise equivalence)*

(S∘7) *If $s \in \mathcal{S}$ and $Y \subseteq \mathcal{P}(\mathcal{S})$, then $\bigcap_{B \in Y} (\{s\} \circ_{\mathcal{S}} B) \subseteq \{s\} \circ_{\mathcal{S}} (\bigcup Y)$ (pointwise inclusion)*

This definition not only adapts the CL update from [8] to the structure context. It also deals with the infinite case by allowing the empty set as an answer. Recall that both update and CL update are originally defined over finite worlds and have a faithful assignment characterization for minimal change using partial orders (see [11]), hence there is always a minimal element. In the AWS framework, update is generalized to the infinite case by working with well-founded partial orders, preserving this idea. However, partial orders typically used in a repair context are not, in general, well-founded. This is compatible with the lack of a consistency postulate. This is why we added **(S∘4)** and **(S∘6)** to recover the behavior of the minimal set in a finite context. These are the main adaptations from [8], apart from writing the operator in terms of models. To reproduce the representation theorem of CL update, we have to define credible faithful assignments.

Definition 3.2. *A credible faithful assignment is a mapping associating each $s \in \mathcal{S}$ with a pair $(C_s, \leq_s)$ where $\{s\} \subseteq C_s \subseteq \mathcal{S}$, $\leq_s$ is a partial order over C_s and $s \leq_s s'$, $s' \not\leq_s s$ for every $s \neq s' \in C_s$. Given a partial order $\leq$ over $A \subseteq \mathcal{S}$, we note $\min(B, \leq) = \{s \in B \mid \forall s' \in B \setminus \{s\},\, s' \not\leq s\}$ for all $B \subseteq A$.*

What follows is the representation theorem for CL $\mathcal{S}$-update operators. Recall that both postulates and the assignment perceive structures as elements of sets, i.e., this is still the AWS framework, where worlds are associated with structures.

Theorem 3.3 (Credibility-limited $\mathcal{S}$-Update Representation Theorem)**.** *An operator $\circ_{\mathcal{S}}$ is a CL $\mathcal{S}$-update iff there is a credible faithful assignment $s \mapsto (C_s, \leq_s)$ such that $\{s\} \circ_{\mathcal{S}} B = \min(B \cap C_s, \leq_s)$, where $C_s = \{z \in \mathcal{S} \mid \{s\} \circ_{\mathcal{S}} \{z\} = \{z\}\}$, and $z \leq_s z'$ iff $\{s\} \circ_{\mathcal{S}} \{z, z'\} = \{z\}$.*

To contextualize Theorem 3.3 with the notion of repairing, consider the following framework taken from [1]. In a relational first-order language setting, given a structure s, let $\Sigma(s)$ be the set of ground atoms satisfied by s.[1] We say that s' is a ***substructure*** of s (or analogously, s is a ***superstructure*** of s'), noted $s' \subseteq s$, if the active domain of s' is contained in the active domain of s and $\Sigma(s') \subseteq \Sigma(s)$.

Definition 3.4. *The* **distance** *between structures s and s' is the symmetric difference:*

$$\Delta(s, s') = (\Sigma(s) - \Sigma(s')) \cup (\Sigma(s') - \Sigma(s)).$$

Given structures s, s', s'', we write $s' \prec_s s''$ if $\Delta(s, s') \subseteq \Delta(s, s'')$. For a given set of formulas Γ, we say s' is a **distance repair** *of (s, Γ) if s' is $\prec_s$-minimal in $\mathbb{W}(\Gamma)$. Moreover, we say it is a:*

subset repair *if s' is $\prec_s$-minimal over the set of substructures of s;*

superset repair *if s' is $\prec_s$-minimal over the set of superstructures of s.*

Given a CL $\mathcal{S}$-update, we represent the set of repairs of a structure s w.r.t a constraint Γ of formulas as the result of considering $\{s\}$ in the first parameter, and $\mathbb{W}(\Gamma)$ in the second parameter. However, for the partial order and extra-logical constraints, such as working with subset or superset repairs, we must consider extra postulates that fix the definition of C_s and $\leq_s$ that appear in Theorem 3.3.

[1] In database theory terminology, s is an instance of a database schema. In Knowledge Representation and Reasoning (KRR) terminology, $\Sigma(s)$ is an ABox and s an interpretation that satisfies $\Sigma(s)$.

(SR∘1) $\{s\} \circ_{\mathcal{S}} \{s'\} \neq \emptyset$ (total success);

(SR∘2) If $s' \prec_s s''$, then $\{s\} \circ_{\mathcal{S}} \{s', s''\} = \{s'\}$ (minimality)

(SR∘3) $s' \subseteq s$ iff $\{s\} \circ_{\mathcal{S}} \{s'\} = \{s'\}$ (subset success);

(SR∘4) If $s'' \subsetneq s' \subseteq s$, then $\{s\} \circ_{\mathcal{S}} \{s', s''\} = \{s'\}$ (subset minimality)

(SR∘5) $s \subseteq s'$ iff $\{s\} \circ_{\mathcal{S}} \{s'\} = \{s'\}$ (superset success);

(SR∘6) If $s \subseteq s' \subsetneq s''$, then $\{s\} \circ_{\mathcal{S}} \{s', s''\} = \{s'\}$ (superset minimality)

Postulate **(SR∘1)** ensures all the structures belonging to $\mathbb{W}(\Gamma)$ are valid candidates as repair (i.e. $C_s = \mathcal{S}$), while **(SR∘2)** codifies the partial order $\prec_s$ to the one related to the operator. Similarly, postulate **(SR∘3)** associates C_s with the set of subset repairs, restricting the candidates to this kind of structure, while postulate **(SR∘4)** associates the substructure relation ($\subseteq$) with $\leq_s$. Analogously, C_s is the set of superset repairs when postulate **(SR∘5)** holds, and **(SR∘6)** links the superset inclusion ($\supseteq$) with $\leq_s$. By combining any pair of these postulates with **(S∘1)**∼**(S∘7)**, we characterize a unique CL $\mathcal{S}$-update operator and axiomatically represent a distance, subset, or superset repair, respectively.

3.1. Repairing inconsistencies over graphs

We will see now some examples of logical languages to reason over graph-like structures, which can be characterized by the belief change operator $\circ_{\mathcal{S}}$. Consider an infinite countable set $\mathsf{N_I}$ of constants, and countable sets $\mathsf{N_C}$ and $\mathsf{N_R}$ of unary and binary relation symbols, respectively.[2] A ***graph-like structure*** (or simply, a ***graph***) G is an interpretation over a finite signature contained in $\mathsf{N_I} \cup \mathsf{N_C} \cup \mathsf{N_R}$. For some languages, the semantics is defined locally, and certain properties are only verified over distinguished nodes of the graph, for which we define a ***pointed graph*** as a pair (G, w), where G is a graph and w is a node of G. The following example shows some particular (pointed) graphs and some path properties expressed in first-order logic.

Example 3.5. *Consider the following constraint sets that can be interpreted over graph-like structures:* $\Gamma = \{\forall x.(\exists y.t(x,y) \Rightarrow \exists zw.r(x,z) \wedge t(z,w))\}$ *and* $\Gamma' = \{\exists y.t(x,y) \Rightarrow \exists zw.r(x,z) \wedge t(z,w)\}$. *These formulas state that if a node has an outgoing edge with label* t *then it is the starting node of a path with label* rt. *Notice that* Γ' *requires an interpretation for the free variable* x, *which means it must be interpreted over pointed graphs. The following figure shows some (pointed) graphs and their relations with* Γ *and* Γ'.

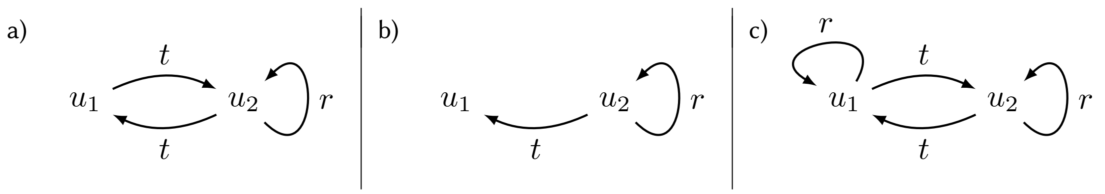

Figure 1: a) A graph structure G, which is inconsistent with respect to Γ, since the node u_1 falsifies the constraint. b) A subset repair G_1 of (G, Γ). c) A superset repair G_2 of (G, Γ). Notice that all these cases can become examples on pointed graphs by considering the structures (G, u_1), (G_1, u_1), (G_2, u_1), and their relation with Γ'.

Unlike relational database theory (closely tied to the SQL framework), there is no standardized language to reason over graph-like structures, at least not one universally agreed upon by researchers. Instead, several families of languages are used across different communities. For instance, the ISO language GQL manages Property Graphs through PG-keys [12]; the W3C standard SHACL (Shapes Constraint Language) validates RDF graphs against specific conditions [13]; and various logical languages have been developed to theoretically formalize industrial languages, some of which we will discuss next and show how these fall under the scope of Definition 2.1.

[2] In KRR terminology, elements of $\mathsf{N_C}$ and $\mathsf{N_R}$ are called ***concept names*** and ***role names***, respectively.

Guarded Negation First Order Logic (GNFO): This first-order logic fragment is given by the following grammar:

$$\varphi, \psi ::= P(\bar{x}) \mid x = y \mid \exists x.\varphi \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid P(\bar{x}\bar{y}) \wedge \neg\varphi(\bar{y}), \quad (2)$$

where $P \in \mathsf{N_C} \cup \mathsf{N_R}$ and $\bar{x}, \bar{y}$ are tuples of first-order variables. The semantics is given by relational models over the signature $\mathsf{N_C} \cup \mathsf{N_R}$, that is, $\mathcal{S}$ is the set of graphs, and the consistency notion we consider is upon sets of sentences[3], that is, if $G \in \mathcal{S}$ and $\Gamma \subseteq$ GNFO is a set of sentences, then $G \models \Gamma$ is defined in the classical sense. GNFO was defined in [14] to extend basic modal logic to a robust fragment of first-order logic by preserving good computational and model properties. GNFO captures many classical database integrity constraints, such as tuple-generating dependencies (TGDs) and inclusion dependencies, and has been treated as a constraint language in [15, 16] to study the problem of consistent query answers when repairing an inconsistency involves adding information.

Propositional Dynamic Logic (PDL): Another generalization of basic modal logic was proposed in [17] by considering diamond symbols constructed via regular language operations. Precisely, PDL is the language defined by the following mutually recursive grammar:

$$\varphi, \psi ::= p \mid \neg\varphi \mid \varphi \wedge \psi \mid \langle\alpha\rangle\varphi$$
$$\alpha, \beta ::= \varepsilon \mid r \mid \alpha \circ \beta \mid \alpha \cup \beta \mid \alpha^* \mid \varphi?$$

where $p \in \mathsf{N_C}$ and $r \in \mathsf{N_R}$, and expressions like φ are called formulas. As in basic modal logic, PDL semantics is given by Kripke models, which can be seen as pointed graphs. Although languages with path patterns and regular operation syntax are usually taken as query languages, in some works such as [18, 19, 16], this expressive behavior is considered to establish *path constraint* over graphs. For instance, in [18], $\mathcal{S}$ represents the set of pointed graphs, and given $(G, w) \in \mathcal{S}$ and $\Gamma \subseteq$ PDL a set of formulas, $G, w \models \Gamma$ if every formula is valid in w. On the other hand, in [19], the satisfiability of a formula is given in a universal sense, that is, $\mathcal{S}$ is the set of graphs and $G \models \Gamma$ if every expression in Γ is valid in every node of G.

Example 3.5 can be adapted to the GNFO and PDL frameworks, showing a situation of repair in both settings. The works [19, 16] are dedicated to analyzing the complexity of computing a repair of an inconsistent graph database, while our proposal extends the notion of repairing into the belief change theory, following the ideas of [7, 4, 5].

4. Conclusions and Future Work

In this work, we have introduced an abstract framework together with a family of belief change operators that capture a notion of repair based on partial orders and a designated set of admissible outcomes as extra-logical constraints. The framework provides a uniform treatment of different repair strategies, and we have illustrated how concrete instances recover well-known approaches, such as distance-based, subset, and superset repair.

As future work, the connection with Formal Concept Analysis deserves further investigation, as it may offer an alternative characterization of the framework and its associated repair operators [20, 21]. Moreover, the framework can be extended by incorporating selection functions [5, 6]. This generalization may provide a more flexible mechanism for characterizing repairs and allow us to capture not only additional notions of admissible repair considered in the literature [3, 4, 22], but also ontology repairs, which are usually connected to contraction operators [23]. We also believe it can be used to develop a framework that simultaneously incorporates the impositions given by a set of integrity constraints and preserves the answers of a given set of queries, also known in the literature as Consistent Query Answering [22]. Lastly, exploring postulate-based characterizations that uniquely determine belief change operators could also be put in correspondence with [24].

[3]GNFO formulas with no free variables.

Acknowledgments

Grimaldi acknowledges partial support of the National Research Foundation of South Africa (REFERENCE NO: SAI240823262612) and Argentinean project UBA-CyT-20020190100021BA.

References

[1] M. Arenas, L. Bertossi, J. Chomicki, Consistent query answers in inconsistent databases, in: Proceedings of the Eighteenth ACM SIGMOD-SIGACT-SIGART Symposium on Principles of Database Systems, PODS '99, Association for Computing Machinery, New York, NY, USA, 1999, p. 68–79. doi:10.1145/303976.303983.

[2] M. Arenas, L. E. Bertossi, J. Chomicki, Answer sets for consistent query answering in inconsistent databases, CoRR cs.DB/0207094 (2002). URL: https://arxiv.org/abs/cs/0207094.

[3] F. Afrati, P. Kolaitis, Repair checking in inconsistent databases: Algorithms and complexity, in: Proceedings of the 12th International Conference on Database Theory, volume 361 of *ICDT '09*, Association for Computing Machinery, New York, NY, USA, 2009, pp. 31–41. doi:10.1145/1514894.1514899.

[4] P. T. Guerra, R. Wassermann, Two agm-style characterizations of model repair, Annals of Mathematics and Artificial Intelligence 87 (2019) 233–257.

[5] D. Grimaldi, Estudio lógico-matemático de una familia de operadores de actualización no-priorizados de bases de conocimiento, Ph.D. thesis, Universidad de Buenos Aires, 2025.

[6] C. E. Alchourrón, P. Gärdenfors, D. Makinson, On the logic of theory change: Partial meet contraction and revision functions, The Journal of Symbolic Logic 50 (1985) 510–530.

[7] Y. Zhang, Y. Ding, Ctl model update for system modifications, J. Artif. Int. Res. 31 (2008) 113–155.

[8] E. Fermé, S. Konieczny, R. Pino Pérez, N. Schwind, Credible Models of Belief Update, in: Proceedings of the 20th International Conference on Principles of Knowledge Representation and Reasoning, 2023, pp. 252–261. doi:10.24963/kr.2023/25.

[9] B. Ganter, R. Wille, Formal Concept Analysis - Mathematical Foundations, Second Editiontion, Springer, 2024. doi:10.1007/978-3-031-63422-2.

[10] S. Lindström, A semantic approach to nonmonotonic reasoning: Inference operations and choice, Theoria 88 (2022) 494–528. doi:https://doi.org/10.1111/theo.12405.

[11] H. Katzuno, A. Mendelson, On the difference between update a knowledge base and revising it, in: in Proceedings of the Second International Conference on Principles of Knowledge Representation and Reasoning, 1991, pp. 387–394.

[12] R. Angles, A. Bonifati, S. Dumbrava, G. Fletcher, A. Green, J. Hidders, B. Li, L. Libkin, V. Marsault, W. Martens, F. Murlak, S. Plantikow, O. Savkovic, M. Schmidt, J. Sequeda, S. Staworko, D. Tomaszuk, H. Voigt, D. Vrgoc, M. Wu, D. Zivkovic, Pg-schema: Schemas for property graphs, Proc. ACM Manag. Data 1 (2023). doi:10.1145/3589778.

[13] H. Knublauch, D. Kontokostas, Shacl use cases and requirements, W3C Working Group Note, 2017. URL: https://www.w3.org/TR/2017/NOTE-shacl-ucr-20170720/.

[14] V. Bárány, B. T. Cate, L. Segoufin, Guarded negation, J. ACM 62 (2015). doi:10.1145/2701414.

[15] D. Figueira, S. Figueira, E. Pin Baque, Finite Controllability for Ontology-Mediated Query Answering of CRPQ, in: Proceedings of the 17th International Conference on Principles of Knowledge Representation and Reasoning, 2020, pp. 381–391. doi:10.24963/kr.2020/39.

[16] E. Pin, Lógicas para razonar sobre grafos con datos, Ph.D. thesis, Universidad de Buenos Aires, 2025.

[17] M. J. Fischer, R. E. Ladner, Propositional dynamic logic of regular programs, Journal of Computer and System Sciences 18 (1979) 194–211. doi:https://doi.org/10.1016/0022-0000(79)90046-1.

[18] N. Alechina, S. Demri, M. de Rijke, A modal perspective on path constraints, Journal of Logic and Computation 13 (2003) 939–956. doi:10.1093/logcom/13.6.939.

[19] S. Abriola, M. V. Martínez, N. Pardal, S. Cifuentes, E. Pin Baque, On the Complexity of Finding Set Repairs for Data-Graphs, J. Artif. Int. Res. 76 (2023). doi:10.1613/jair.1.13994.

[20] F. Baader, B. Sertkaya, Applying formal concept analysis to description logics, volume 2961, 2004, pp. 261–286. doi:10.1007/978-3-540-24651-0_24.

[21] F. Baader, B. Ganter, B. Sertkaya, U. Sattler, Completing description logic knowledge bases using formal concept analysis., 2007, pp. 230–235.

[22] N. Fröhlich, A. Meier, N. Pardal, J. Virtema, A logic-based framework for database repairs, in: Proceedings of the 22nd International Conference on Principles of Knowledge Representation and Reasoning, KR '25, 2025. URL: https://doi.org/10.24963/kr.2025/32. doi:10.24963/kr.2025/32.

[23] F. Baader, R. Wassermann, Contractions based on optimal repairs (extended abstract), in: J. Kwok (Ed.), Proceedings of the Thirty-Fourth International Joint Conference on Artificial Intelligence, IJCAI-25, International Joint Conferences on Artificial Intelligence Organization, 2025, pp. 10852–10857. doi:10.24963/ijcai.2025/1204, sister Conferences Best Papers.

[24] C. A. Deagustini, M. V. Martinez, M. A. Falappa, G. R. Simari, Belief base contraction by belief accrual, Artificial Intelligence 275 (2019) 78–103. doi:https://doi.org/10.1016/j.artint.2019.05.002.

Appendix: Proofs

Proof of Theorem 2.2. Let us first check $(\mathbb{W}, \mathbb{T})$ is an antitone Galois connection:

$$A \subseteq \mathbb{W}(\Gamma) \quad \text{iff} \quad s \models \gamma \text{ for every } s \in A\,,\ \gamma \in \Gamma \quad \text{iff} \quad \Gamma \subseteq \bigcap_{s\in A} T_s = \mathbb{T}(A)$$

Now we continue with the list of properties. From the Antitone Galois Connection, it is possible to show that the compositions are Tarskian operators. First, note that $\mathbb{W}$ and $\mathbb{T}$ are order-reversing due to their definition using intersection. Thus, both compositions satisfy **Monotonicity** by the Galois connection. The **Inclusion** property, for example, $A \subseteq Cl(A)$, is equivalent to showing $\mathbb{T}(A) \subseteq \mathbb{T}(A)$ due to Equation 1, which trivially holds. Analogously for Cn. Lastly, **Idempotence**, for example $Cl(Cl(A)) = Cl(A)$, is deduced by the **Inclusion** property of Cl, $Cl(A) \subseteq Cl(Cl(A))$, and the **Inclusion** property of Cn: when applying $\mathbb{T}$ to $\mathbb{W}(A) \subseteq Cn(\mathbb{W}(A))$, we have $Cl(Cl(A)) \subseteq Cl(A)$ since $\mathbb{W}(Cl(A)) = Cn(\mathbb{W}(A))$ by definition. Similarly, we have $\mathbb{W}(A) \subseteq Cn(\mathbb{W}(A))$ and $\mathbb{W}(Cl(A)) \subseteq \mathbb{W}(A)$, thus we $\mathbb{W}(Cl(A)) = Cn(\mathbb{W}(A)) = \mathbb{W}(A)$. The same happens to Cn.

Lastly, we show $\mathbb{T}$ and $\mathbb{W}$ are lattice isomorphisms between $\mathcal{K}$ and $\mathcal{C}$. Note $\mathbb{T}$ is a bijection from $\mathcal{C}$ to $\mathcal{K}$. If $K \in \mathcal{K}$, then $A = \mathbb{W}(K)$ satisfies $\mathbb{T}(A) = K$. Therefore, $\mathbb{T}$ is surjective. Let A, $A' \in \mathcal{C}$ such that $\mathbb{T}(A) = \mathbb{T}(A')$. Then, by applying $\mathbb{W}$, we have $A = Cl(A) = Cl(A') = A'$. Thus $\mathbb{T}$ is injective. Also, $\mathbb{W}$ is the inverse of $\mathbb{T}$, since $\mathbb{W}(\mathbb{T}(A)) = Cl(A) = A$ for every $A \in \mathcal{C}$.

To show that they preserve the lattice structure, note $\mathbb{T}(\emptyset) = \mathbb{L}$ and $\mathbb{W}(\emptyset) = \mathcal{S}$, while $\mathbb{T}(\mathcal{S}) = Cn(\emptyset)$ and $\mathbb{W}(\mathbb{L}) = Cl(\emptyset)$. This connects the least and the greatest elements of each lattice. Take now $Y \subseteq \mathcal{P}(\mathcal{S})$ and $X \subseteq \mathcal{P}(\mathbb{L})$.

$$\bigcap \mathbb{T}(Y) = \bigcap_{A\in Y} \bigcap_{s\in A} T_s = \bigcap_{s\in \bigcup_s Y} T_s = \mathbb{T}\left(\bigcup Y\right)$$

$$\bigcap \mathbb{W}(X) = \bigcap_{\Gamma\in X} \bigcap_{\psi\in\Gamma} W_\psi = \bigcap_{\psi\in \bigcup X} W_\psi = \mathbb{W}\left(\bigcup X\right)$$

Therefore, if $Y \subseteq \mathcal{C}$ and $X_Y = \mathbb{T}(Y) \subseteq \mathcal{K}$, and analogously, if $X \subseteq \mathcal{K}$ and $Y_X = \mathbb{T}(X) \subseteq \mathcal{C}$, then:

$$\bigcap Y = \bigcap \mathbb{W}(X_Y) = \mathbb{W}\left(\bigcup X_Y\right) = \mathbb{W}\left(Cn\left(\bigcup X_Y\right)\right) = \mathbb{W}\left(\bigoplus X_Y\right)$$

$$\bigcap X = \bigcap \mathbb{T}(Y_X) = \mathbb{T}\left(\bigcup_s Y_X\right) = \mathbb{T}\left(Cl\left(\bigcup Y_X\right)\right) = \mathbb{T}\left(\bigoplus Y_X\right)$$

By the bijection, we conclude that $\bigcap$ is preserved by $\bigoplus$ for both lattices. $\square$

Proof of Theorem 3.3. **From postulates to construction:** Given some $s \in A$, define:

- $C_s = \{r \in \mathcal{S} \mid \{s\} \circ_{\mathcal{S}} \{r\} = \{r\}\}$;
- $r \leq_s r'$ iff $\{s\} \circ_{\mathcal{S}} \{r, r'\} = \{r\}$.

By **(S∘2)**, $s \leq_s r$, $r \not\leq_s s$ for every $s \neq r \in C_s$. Thus, we want to show that $\leq_s$ is a partial order over C_s. By construction, $\leq_s$ is antisymmetric and reflexive over C_s. For transitivity, assume $u, v, w \in C_s$ such that $u \leq_s v$ and $v \leq_s w$. If $u = v$, then $u \leq_s w$ by definition. Otherwise, since $u \neq v$, we also have $v \neq s$ and $w \neq s$. Let us now analyze the result of $\{s\} \circ_{\mathcal{S}} \{u, v, w\}$. Due to **(S∘4)** and **(S∘5)**, we have:

- $\{s\} \circ_{\mathcal{S}} \{u, v, w\} \neq \emptyset$ since $\{s\} \circ_{\mathcal{S}} \{u\} = \{u\} \neq \emptyset$;
- $(\{s\} \circ_{\mathcal{S}} \{u, v, w\}) \cap \{u, v\} \subseteq \{s\} \circ_{\mathcal{S}} \{u, v\} = \{u\}$, hence $v \notin \{s\} \circ_{\mathcal{S}} \{u, v, w\}$;
- $(\{s\} \circ_{\mathcal{S}} \{u, v, w\}) \cap \{v, w\} \subseteq \{s\} \circ_{\mathcal{S}} \{v, w\} = \{v\}$, hence $w \notin \{s\} \circ_w \{u, v, w\}$.

Therefore, $\{s\} \circ_{\mathcal{S}} \{u, v, w\} = \{u\}$ by **(S∘1)**. Since $\emptyset \neq \{s\} \circ_{\mathcal{S}} \{u, v, w\} \subseteq \{u, w\} \subseteq \{u, v, w\}$, we can apply **(S∘6)** to deduce $\{s\} \circ_{\mathcal{S}} \{u, w\} = \{u\}$. This means $u \leq_s w$, allowing us to conclude $\leq_s$ is a partial order. Finally, since this construction was made for an arbitrary $s \in \mathcal{S}$, what we have is a credible faithful assignment.

Let us check that given this faithful assignment, the equality $\{s\} \circ_{\mathcal{S}} B = \min(B \cap C_s, \leq_s)$ is valid. First of all, suppose $\{s\} \circ_{\mathcal{S}} B \neq \emptyset$, and note that $\{s\} \circ_{\mathcal{S}} B \subseteq B \cap C_s$: if $m \in \{s\} \circ_{\mathcal{S}} B$, then $m \in B$ by **(S∘1)**, and by **(S∘5)** we have $m \in (\{s\} \circ_{\mathcal{S}} B) \cap \{m\} \subseteq \{s\} \circ_{\mathcal{S}} \{m\}$; thus $\{s\} \circ_{\mathcal{S}} \{m\} = \{m\}$ by **(S∘1)**. Due to $\emptyset \neq \{s\} \circ_{\mathcal{S}} B \subseteq B \cap C_s \subseteq B$, we can deduce $\{s\} \circ_{\mathcal{S}} B = \{s\} \circ_{\mathcal{S}} (B \cap C_s)$ by **(S∘6)**. Consider now $m \in \{s\} \circ_{\mathcal{S}} (B \cap C_s)$ and $r \in B \cap C_g$ such that $m \neq r$. By **(S∘5)** we have $m \in (\{s\} \circ_{\mathcal{S}} B) \cap \{m, r\} \subseteq \{s\} \circ_{\mathcal{S}} \{m, r\}$. Which means, due to **(S∘1)**, that $\{m\} \subseteq \{s\} \circ_{\mathcal{S}} \{m, r\} \subseteq \{m, r\}$. Hence, $r \not\leq_s m$, and thus $m \in \min(B \cap C_s, \leq_s)$. Therefore, $\{s\} \circ_{\mathcal{S}} B \subseteq \min(B \cap C_s, \leq_s)$, since this inclusion trivially holds when $\{s\} \circ_{\mathcal{S}} B = \emptyset$.

Assume now $\min(B \cap C_s, \leq_s) \neq \emptyset$ and take $m \in \min(B \cap C_s, \leq_s)$. Then, for every $r \in B \cap C_s$ we have $\{m\} \subseteq \{s\} \circ_{\mathcal{S}} \{m, r\} \subseteq \{m, r\}$. Also, $\{s\} \circ_{\mathcal{S}} \{m, r\} = \{m\}$ for every $r \in B \setminus C_s$ due to:

- $\{s\} \circ_{\mathcal{S}} \{m, r\} \neq \emptyset$ by **(S∘4)** since $\{s\} \circ_{\mathcal{S}} \{m\} = \{m\} \neq \emptyset$;
- $(\{s\} \circ_{\mathcal{S}} \{m, r\}) \cap \{r\} \subseteq \{s\} \circ_{\mathcal{S}} \{r\} = \emptyset$ by **(S∘5)**, hence $r \notin \{s\} \circ_{\mathcal{S}} \{m, r\}$;
- $\{s\} \circ_{\mathcal{S}} \{m, r\} \subseteq \{m, r\}$ by **(S∘1)**.

Therefore, $\{m\} \subseteq \{s\} \circ_{\mathcal{S}} \{m, r\} \subseteq \{m, r\}$ for every $r \in B$. Now we can apply **(S∘7)** to deduce:

$$m \in \bigcap_{r \in B} \{s\} \circ_{\mathcal{S}} \{m, r\} \subseteq \{s\} \circ_{\mathcal{S}} B$$

Therefore, $\min(B \cap C_s, \leq_s) \subseteq \{s\} \circ_{\mathcal{S}} B$. This ends the proof of $\min(B \cap C_s, \leq_s) = \{s\} \circ_{\mathcal{S}} B$.

From construction to postulates: Given a credible faithful assignment, we define $\circ_{\mathcal{S}}$ as follows:

$$A \circ_{\mathcal{S}} B = \bigcup_{s \in A} \min(B \cap C_s, \leq_s)$$

This construction clearly satisfies **(S∘1)** and **(S∘3)** by definition. Also, if $A \subseteq B$, then $s \in B \cap C_s$ for every $s \in A$, thus $\min(B \cap C_s, \leq_s) = \{s\}$. Hence, $A \circ_{\mathcal{S}} B = A$, i.e. $\circ_{\mathcal{S}}$ satisfies **(S∘2)**. For the rest of the postulates, we need a more developed deduction:

(S∘4) Take $A, B, C \subseteq \mathcal{S}$ such that $A \circ_{\mathcal{S}} B \neq \emptyset$ and $B \subseteq C$ are finite sets. Consider $s \in A$ such that $\min(B \cap C_s, \leq_s) \neq \emptyset$, which exists by hypothesis. Then, $\emptyset \neq \min(B \cap C_s, \leq_s) \subseteq B \cap C_s \subseteq C \cap C_s$ and these are all finite sets. Therefore, $\emptyset \neq \min(C \cap C_s, \leq_s)$. We conclude $A \circ_{\mathcal{S}} C \neq \emptyset$, since $A \circ_{\mathcal{S}} C = \bigcup_{s \in A} \min(C \cap C_s, \leq_s)$.

(S∘5) Let $s \in \mathcal{S}$. Given $m \in \min(B \cap C_s, \leq_s) \cap C$, we deduce that $m \in B \cap C_s \cap C$. Take $r \in B \cap C_s \cap C$, $r \neq m$. Then $r \not\leq_s m$ since $m \in \min(B \cap C_s, \leq_s)$ and $r \in B \cap C_s$. Hence $m \in \min(B \cap C_s \cap C, \leq_s)$. Therefore, $\min(B \cap C_s, \leq_s) \cap C \subseteq \min(B \cap C_s \cap C, \leq_s)$, and **(S∘5)** holds by construction.

(S∘6) Let $s \in \mathcal{S}$ and $\emptyset \neq \{s\} \circ_{\mathcal{S}} B \subseteq C \subseteq B$. By construction, we have $\emptyset \neq \min(B \cap C_s, \leq_s) \subseteq C \subseteq B$. Hence, $\emptyset \neq \min(B \cap C_s, \leq_s) \subseteq C \cap C_s \subseteq B \cap C_s$, since $\min(B \cap C_s, \leq_s) \subseteq C_s$. Take some $m \in \min(B \cap C_s, \leq_s)$, and note that $m \in C \cap C_s$. Since $C \cap C_s \subseteq B \cap C_s$, then $r \not\leq_s m$ for every $r \in C \cap C_s$. Therefore, $m \in \min(C \cap C_s, \leq_s)$.

For the other inclusion, take $m \in \min(C \cap C_s, \leq_s)$ and assume $r \leq_s m$ for some $r \in B \cap C_s$, $r \neq m$. By definition, $r \not\leq_s m'$ for every $m' \in \min(B \cap C_s, \leq_s)$. Hence, either there is some $m' \in \min(B \cap C_s, \leq_s)$ such that $m' \leq_s r$ or every $m' \in \min(B \cap C_g, \leq_g)$ does not relate to r. The first case implies that $m' \leq_s m$, $m' \neq m$ by transitivity, and the latter implies that $r \in \min(B \cap C_s, \leq_s)$. In any case, we conclude there is some $m' \in \min(B \cap C_s, \leq_s)$ such that $m' \leq_s m$, $m' \neq m$. But $m' \in C \cap C_s$, since $\min(B \cap C_s, \leq_s) \subseteq C \cap C_s$. Hence, $m' \not\leq_s m$ since $m \in \min(C \cap C_s, \leq_s)$, a contradiction. Therefore, there $r \not\leq_s m$ for every $r \in B \cap C_s$, $r \neq m$, i.e., $m \in \min(C \cap C_s, \leq_s)$. We conclude $\{s\} \circ_{\mathcal{S}} B = \min(B \cap C_s, \leq_s) = \min(C \cap C_s, \leq_s) = \{s\} \circ_{\mathcal{S}} C$.

(S∘7) Let $s \in \mathcal{S}$ and $Y \subseteq \mathcal{P}(\mathcal{S})$, and take $m \in \bigcap_{B \in Y} \{s\} \circ_{\mathcal{S}} B = \bigcap_{B \in Y} \min(B \cap C_s, \leq_s)$. Consider an arbitrary $r \in C_s \cap \bigcup Y$ such that $m \neq r$. Then, there is some $B_r \in Y$ such that $r \in B_r$. Since $m \in \min(B_r \cap C_s, \leq_s)$, we have $r \not\leq_s m$. Hence, $m \in \min(C_s \cap \bigcup Y, \leq_s) = \{s\} \circ_{\mathcal{S}} (\bigcup Y)$ since r was arbitrarily chosen. Therefore, $\bigcap_{B \in Y} \{s\} \circ_{\mathcal{S}} B \subseteq \{s\} \circ_{\mathcal{S}} (\bigcup Y)$.

□