



Model Repairing and Belief Change Operators

Daniel Grimaldi^{1,2,3*} ; E. Pin Baque⁴

{dgrimaldi, epin}@dc.uba.ar

¹ UCT and CAIR, ² DC-FCEyN-UBA, ³ ICC-UBA-CONICET and ⁴ DM-FCEyN-UBA.



Introduction

Classical approaches in database theory and consistent query answering provide a standardized account of repairs based on minimality criteria [2]. Inspired by recent work linking repairs to belief change [9], we generalize the idea to a framework built on the Abstract Worlds Semantics framework [8], which captures the semantics of AGM theory [1]. We define a family of belief change operators based on [5]. This setting allows the operator to uniformly capture distance-based, subset, and superset repairing. Moreover, this abstract formulation admits an alternative reading in terms of Formal Concept Analysis [7], bridging database repair and knowledge representation.

The General Setup

Definition 1. We say a language $\mathbb{L}$, a set of structures $\mathcal{S}$, and a relation $\models \subseteq \mathcal{S} \times \mathbb{L}$, define a **logic** $\mathcal{L} = \langle \mathbb{L}, \mathcal{S}, \models \rangle$. A logic is always associated with the following functions:

- $\mathbb{W} : \mathcal{P}(\mathbb{L}) \rightarrow \mathcal{P}(\mathcal{S})$ such that $\mathbb{W}(\Gamma) = \bigcap_{\psi \in \Gamma} W_\psi$, where $W_\psi = \{s \in \mathcal{S} \mid s \models \psi\}$.
- $\mathbb{T} : \mathcal{P}(\mathcal{S}) \rightarrow \mathcal{P}(\mathbb{L})$ such that $\mathbb{T}(A) = \bigcap_{s \in A} T_s$, where $T_s = \{\psi \in \mathbb{L} \mid s \models \psi\}$.
- $Cn = \mathbb{T} \circ \mathbb{W}$ and $Cl = \mathbb{W} \circ \mathbb{T}$.

The previous definition not only connects $\mathbb{L}$ and $\mathcal{S}$ to perceive them as a logic, but it also develops a connection with the AWS framework.

Theorem 2. Let $\mathcal{L} = \langle \mathbb{L}, \mathcal{S}, \models \rangle$. Then, $(\mathbb{W}, \mathbb{T})$ is an antitone Galois connection, i.e.:

$$A \subseteq \mathbb{W}(\Gamma) \quad \text{iff} \quad \Gamma \subseteq \mathbb{T}(A) \quad \forall A \in \mathcal{P}(\mathcal{S}), \Gamma \in \mathcal{P}(\mathbb{L}) \quad (1)$$

Also, the following properties are satisfied:

1. $\mathbb{W}$ and $\mathbb{T}$ are order reversing, and Cn and Cl are Tarskian operators;
2. $\mathbb{W}(\Gamma) = \mathbb{W}(Cn(\Gamma)) = Cl(\mathbb{W}(\Gamma))$ and $\mathbb{T}(A) = Cn(\mathbb{T}(A)) = \mathbb{T}(Cl(A))$;
3. Noting $\oplus$ the closure of the union, $\mathbb{W}$ and $\mathbb{T}$ define isomorphisms between:
the lattice of theories: $(\mathcal{K} = \{K \subseteq \mathbb{L} \mid K = Cn(K)\}, \subseteq, \cap, \oplus, Cn(\emptyset), \mathbb{L})$
the lattice of closed sets: $(\mathcal{C} = \{A \subseteq \mathcal{S} \mid A = Cl(A)\}, \supseteq, \oplus, \cap, \mathcal{S}, Cl(\emptyset))$

This setup can also be understood under Formal Concept Analysis [7], where $\langle \mathcal{S}, \mathbb{L}, \models \rangle$ is known as a formal context, and the pairs (A, K) where $K = \mathbb{T}(A)$ and $A = \mathbb{W}(K)$ are formal concepts.

Credibility-limited $\mathcal{S}$ -Update Operators

We adapt credibility-limited (CL) update operators [5] based on the following:

- (a) having one structure s to repair connects with the local nature of update;
- (b) given $\Gamma \subseteq \mathbb{L}$, we look for the minimal elements of $\mathbb{W}(\Gamma)$ w.r.t $\leq_s$;
- (c) subset or superset structures can be seen as extra-logic constraints.

Definition 3. We say $\circ_{\mathcal{S}} : \mathcal{P}(\mathcal{S}) \times \mathcal{P}(\mathcal{S}) \rightarrow \mathcal{P}(\mathcal{S})$ is a **credibility-limited (CL) $\mathcal{S}$ -update** if it satisfies the following postulates:

- (S $\circ$ 1) $A \circ_{\mathcal{S}} B \subseteq B$
- (S $\circ$ 2) If $A \subseteq B$ then $A \circ_{\mathcal{S}} B = A$
- (S $\circ$ 3) $A \circ_{\mathcal{S}} B = \bigcup_{s \in A} (\{s\} \circ_{\mathcal{S}} B)$
- (S $\circ$ 4) If $A \circ_{\mathcal{S}} B \neq \emptyset$ and $B \subseteq C$ are finite sets, then $A \circ_{\mathcal{S}} C \neq \emptyset$
- (S $\circ$ 5) $(A \circ_{\mathcal{S}} B) \cap C \subseteq A \circ_{\mathcal{S}} (B \cap C)$
- (S $\circ$ 6) If $s \in \mathcal{S}$ and $\emptyset \neq \{s\} \circ_{\mathcal{S}} B \subseteq C \subseteq B$, then $\{s\} \circ_{\mathcal{S}} B = \{s\} \circ_{\mathcal{S}} C$
- (S $\circ$ 7) If $s \in \mathcal{S}$ and $Y \subseteq \mathcal{P}(\mathcal{S})$, then $\bigcap_{B \in Y} (\{s\} \circ_{\mathcal{S}} B) \subseteq \{s\} \circ_{\mathcal{S}} (\bigcup Y)$

Postulates (S $\circ$ 4) and (S $\circ$ 6) capture the typical partial orders used in a repair context. The representation theorem works with credible faithful assignments [5].

Theorem 4. An operator $\circ_{\mathcal{S}}$ is a CL $\mathcal{S}$ -update iff there is a **credible faithful assignment**, i.e. a mapping $s \mapsto (C_s, \leq_s)$ where $\leq_s$ is a partial order over C_s , $s \in C_s \subseteq \mathcal{S}$, and s is the unique minimum element in C_s , such that $\{s\} \circ_{\mathcal{S}} B = \min(B \cap C_s, \leq_s)$, where $z \leq_s z'$ iff $\{s\} \circ_{\mathcal{S}} \{z, z'\} = \{z\}$, and $C_s = \{z \in \mathcal{S} \mid \{s\} \circ_{\mathcal{S}} \{z\} = \{z\}\}$.

Model Repairing as a Belief Change Operator

To contextualize Theorem 4 with the notion of repairing, consider the following framework taken from [2].

Definition 5. The **distance** between structures s and s' is based on the symmetric difference $\Delta(s, s')$. Given structures s, s', s'' , we write $s' \prec_s s''$ iff $\Delta(s, s') \subseteq \Delta(s, s'')$.

For a given set of formulas Γ , we say s' is a **distance repair** of (s, Γ) if s' is $\prec_s$ -minimal in $\mathbb{W}(\Gamma)$. Moreover, we say it is a **subset repair** (resp. **superset repair**) if s' is $\prec_s$ -minimal over the set of substructures (resp. superstructures) of s .

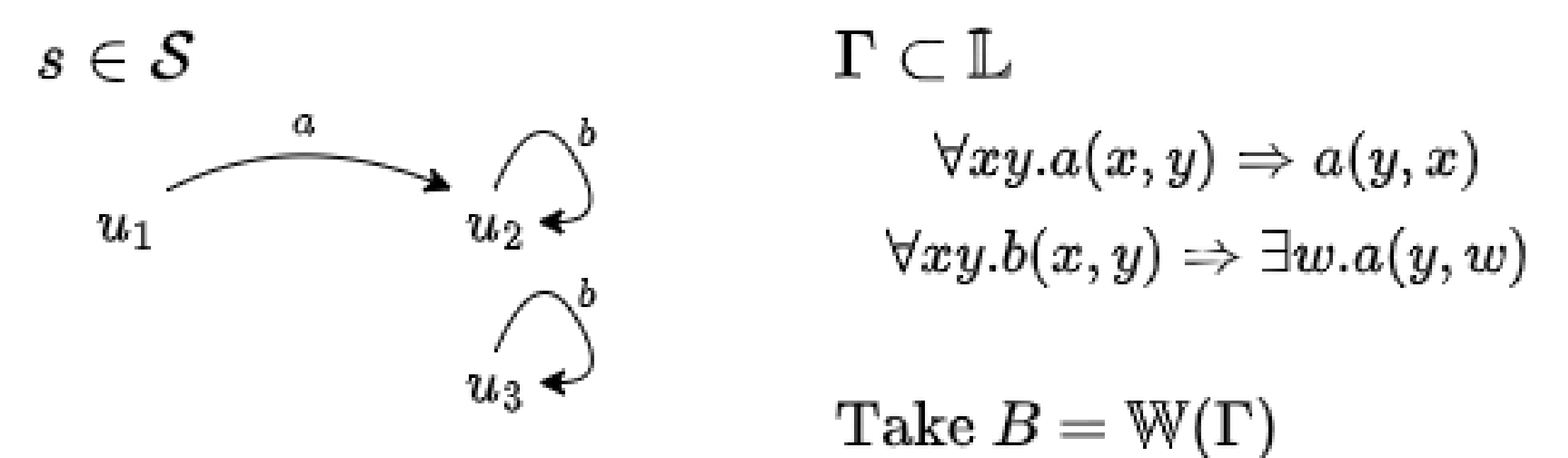
Given a CL $\mathcal{S}$ -update, we represent the set of repairs of a structure s w.r.t a constraint Γ of formulas as the result of considering $\{s\}$ and $\mathbb{W}(\Gamma)$ as parameters. By considering extra postulates that fix the definition of C_s and $\leq_s$ of Theorem 4, we can explicitly associate the operator to distance, subset or superset repairs.

- (SR $\circ$ 1) $\{s\} \circ_{\mathcal{S}} \{s'\} \neq \emptyset$ (total success);
- (SR $\circ$ 2) If $s' \prec_s s''$, then $\{s\} \circ_{\mathcal{S}} \{s', s''\} = \{s'\}$ (minimality)
- (SR $\circ$ 3) $s' \subseteq s$ iff $\{s\} \circ_{\mathcal{S}} \{s'\} = \{s'\}$ (subset success);
- (SR $\circ$ 4) If $s'' \subsetneq s' \subseteq s$, then $\{s\} \circ_{\mathcal{S}} \{s', s''\} = \{s'\}$ (subset minimality)
- (SR $\circ$ 5) $s \subseteq s'$ iff $\{s\} \circ_{\mathcal{S}} \{s'\} = \{s'\}$ (superset success);
- (SR $\circ$ 6) If $s \subseteq s' \subsetneq s''$, then $\{s\} \circ_{\mathcal{S}} \{s', s''\} = \{s'\}$ (superset minimality)

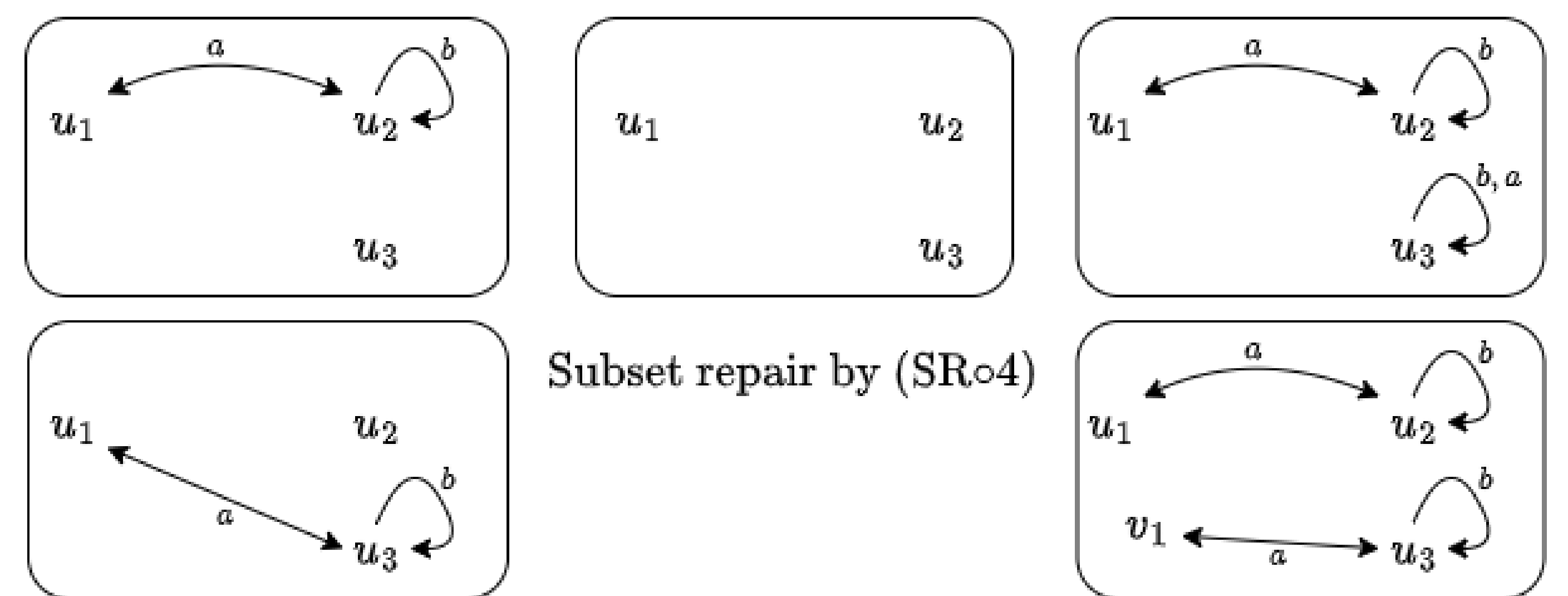
By combining any pair of these postulates with (S $\circ$ 1)~(S $\circ$ 7), we characterize a unique CL $\mathcal{S}$ -update operator and axiomatically represent a distance, subset, or superset repair, respectively.

Repairing inconsistencies over graphs

In a first-order relational setting for graphs, consider the following structure s and set of constraints Γ . Note that $s \not\models \Gamma$ but this inconsistency can be dealt in many ways by considering different approaches. In the context of Theorem 4, the repairing of s with respect to Γ is described as follows:



- (SR $\circ$ 1) $C_s = \mathcal{S}$
- (SR $\circ$ 3) $C_s = \{s' \subseteq s\}$
- (SR $\circ$ 5) $C_s = \{s' \supseteq s\}$



Distance repairs by (SR $\circ$ 2)

Superset repairs by (SR $\circ$ 6)

Conclusions and Future Work

We introduced an abstract framework for database repairs, based on belief change operators, that uniformly captures distance-based, subset, and superset repair, among others. Future directions include investigating the connection with Formal Concept Analysis [3], and extending the framework with selection functions to capture further repair notions and ontology repairs [4]. We also aim to develop a unified approach to Consistent Query Answering that preserves answers under constraints [6].

[1] C. E. Alchourrón et al. "On the logic of theory change: Partial meet contraction and revision functions". In: *The Journal of Symbolic Logic* (1985).
[2] Marcelo Arenas et al. "Consistent query answers in inconsistent databases". In: *Proceedings of the Eighteenth ACM SIGMOD-SIGACT-SIGART Symposium on Principles of Database Systems*. PODS '99. USA, 1999.
[3] Franz Baader et al. "Applying Formal Concept Analysis to Description Logics". In: 2004.
[4] Franz Baader et al. "Contractions Based on Optimal Repairs (Extended Abstract)". In: *Proceedings of the Thirty-Fourth International Joint Conference on Artificial Intelligence, IJCAI-25*. 2025.
[5] Eduardo Fermé et al. "Credible Models of Belief Update". In: *Proceedings of the 20th International Conference on Principles of Knowledge Representation and Reasoning*. 2023.
[6] Nicolas Fröhlich et al. "A logic-based framework for database repairs". In: *Proceedings of the 22nd International Conference on Principles of Knowledge Representation and Reasoning*. KR '25. Melbourne, Australia, 2025.
[7] Bernhard Ganter et al. *Formal Concept Analysis - Mathematical Foundations, Second Edition*. 2024.
[8] Daniel Grimaldi. "Estudio lógico-matemático de una familia de operadores de actualización no-priorizados de bases de conocimiento". PhD diss. Universidad de Buenos Aires, 2025.
[9] Paulo T Guerra et al. "Two AGM-style characterizations of model repair". In: *Annals of Mathematics and Artificial Intelligence* (2019).