

Model Repairing and Belief Change Operators

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Logical approaches to handling INconsistent DAta

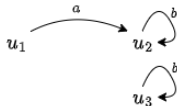
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Model Repairing Example

Idea: minimally modifying a structure so as to satisfy constraints.

$s \in \mathcal{S}$



$\Gamma \subset \mathbb{L}$

$$\forall xy. a(x, y) \Rightarrow a(y, x)$$

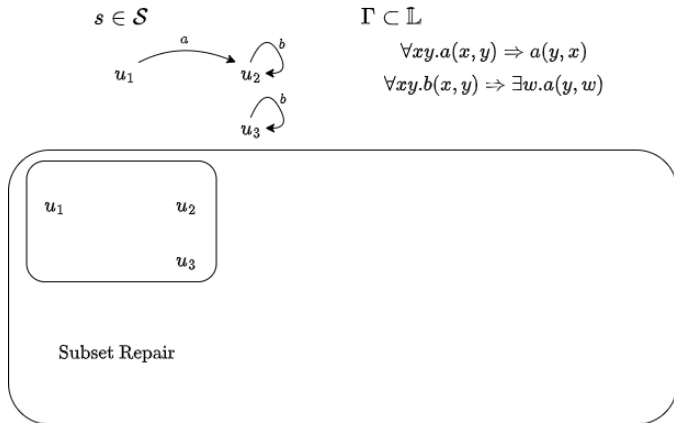
$$\forall xy. b(x, y) \Rightarrow \exists w. a(y, w)$$

$s \not\models \Gamma$

We fix this by looking elements in $\{s' \models \Gamma\}$

Model Repairing Example

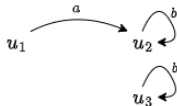
Search restricted to Subset Graphs, based on closeness.



Model Repairing Example

Search restricted to Superset Graphs, based on closeness.

$s \in \mathcal{S}$

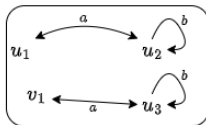
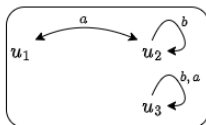


$\Gamma \subset \mathbb{L}$

$$\forall xy. a(x, y) \Rightarrow a(y, x)$$

$$\forall xy. b(x, y) \Rightarrow \exists w. a(y, w)$$

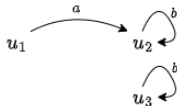
Superset Repair



Model Repairing Example

Search based on a minimality concept.

$s \in \mathcal{S}$

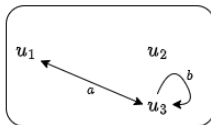
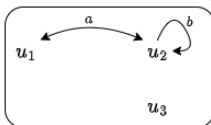


$\Gamma \subset \mathbb{L}$

$$\forall xy. a(x, y) \Rightarrow a(y, x)$$

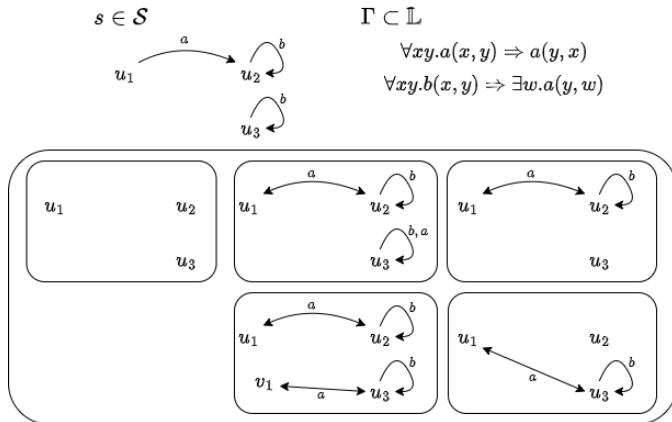
$$\forall xy. b(x, y) \Rightarrow \exists w. a(y, w)$$

Distance Repair



Model Repairing Example

Always restricted search based on a minimality concept.



The General Setup

Given a language $\mathbb{L}$, a set of structures $\mathcal{S}$, and a relation $\models \subseteq \mathcal{S} \times \mathbb{L}$ we say $\mathcal{L} = \langle \mathbb{L}, \mathcal{S}, \models \rangle$ is a **logic**. It is always associated with the functions:

- $\mathbb{W} : \mathcal{P}(\mathbb{L}) \rightarrow \mathcal{P}(\mathcal{S})$ s.t. $\mathbb{W}(\Gamma) = \bigcap_{\psi \in \Gamma} W_\psi$, where $W_\psi = \{s \in \mathcal{S} \mid s \models \psi\}$
- $\mathbb{T} : \mathcal{P}(\mathcal{S}) \rightarrow \mathcal{P}(\mathbb{L})$ s.t. $\mathbb{T}(A) = \bigcap_{s \in A} T_s$, where $T_s = \{\psi \in \mathbb{L} \mid s \models \psi\}$
- $Cn = \mathbb{T} \circ \mathbb{W}$ and $Cl = \mathbb{W} \circ \mathbb{T}$, which are Tarskian operators.

Galois Connection of Tarskian Logics

Let $\mathcal{L} = \langle \mathbb{L}, \mathcal{S}, \models \rangle$, then $(\mathbb{W}, \mathbb{T})$ is an antitone Galois connection, i.e.:

$$A \subseteq \mathbb{W}(\Gamma) \quad \text{if and only if} \quad \Gamma \subseteq \mathbb{T}(A) \quad , \quad \forall A \in \mathcal{P}(\mathcal{S}), \Gamma \in \mathcal{P}(\mathbb{L})$$

Let $\bigoplus$ be the closure of the union, $\mathbb{W}$ and $\mathbb{T}$ define isomorphisms between:

- $(\mathcal{K} = \{K \subseteq \mathbb{L} \mid K = Cn(K)\}, \subseteq, \cap, \bigoplus, Cn(\emptyset), \mathbb{L})$
- $(\mathcal{C} = \{A \subseteq \mathcal{S} \mid A = Cl(A)\}, \supseteq, \bigoplus, \cap, \mathcal{S}, Cl(\emptyset))$

Credibility-limited $\mathcal{S}$ -Update Operators

Based on the general setup and that:

- (a) repairing takes one structure s , linked to the local nature of update;
- (b) given $\Gamma \subseteq \mathbb{L}$, we look for the minimal elements of $\mathbb{W}(\Gamma)$ w.r.t $\leq_s$;
- (c) Subset or superset elements can be seen as extra-logic constraints.

We adapted CL update operators (Fermé et al., 2023), which are defined by credible faithful assignment: $s \mapsto (C_s, \leq_s)$ where $s \in C_s \subseteq \mathcal{S}$, $\leq_s$ is a partial order over C_s , and s is the unique minimum element in C_s .

Credibility-limited $\mathcal{S}$ -Update Representation Theorem

An operator $\circ_{\mathcal{S}}$ is a CL $\mathcal{S}$ -update iff there is a credible faithful assignment $s \mapsto (C_s, \leq_s)$ such that $\{s\} \circ_{\mathcal{S}} B = \min(B \cap C_s, \leq_s)$, where:

- $C_s = \{z \in \mathcal{S} \mid \{s\} \circ_{\mathcal{S}} \{z\} = \{z\}\}$;
- $z \leq_s z'$ iff $\{s\} \circ_{\mathcal{S}} \{z, z'\} = \{z\}$.

We develop an axiomatic characterization of CL $\mathcal{S}$ -update operators.

Thank you!

Have questions? Let's talk at the poster session

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