Errors Need Not Show Up as Inconsistencies

From Error-tolerant Reasoning to Argumentation Frameworks

Franz Baader[1, 2], Oscar Barreca[1, 2]

[1]Technische Universität Dresden

[2]Center for Scalable Data Analytics and Artificial Intelligence (ScaDS.AI) Dresden/Leipzig

Not Every Error Is an Inconsistency

An inconsistent *knowledge base* (KB) is certainly erroneous, but consistent KBs can contain **errors**, too:

$$
\begin{aligned}
\mathrm{FingerAmputation} &\equiv \mathrm{Amputation} \sqcap \exists \mathrm{site}.\mathrm{Finger}_S, \\
\mathrm{HandAmputation} &\equiv \mathrm{Amputation} \sqcap \exists \mathrm{site}.\mathrm{Hand}_S, \\
\mathrm{FingerInjury} &\equiv \mathrm{Injury} \sqcap \exists \mathrm{site}.\mathrm{Finger}_S, \\
\mathrm{HandInjury} &\equiv \mathrm{Injury} \sqcap \exists \mathrm{site}.\mathrm{Hand}_S, \\
\mathrm{Finger}_E &\sqsubseteq \mathrm{Finger}_S, \\
\mathrm{Finger}_P &\sqsubseteq \mathrm{Finger}_S \sqcap \exists \mathrm{part}.\mathrm{Finger}_E, \\
\mathrm{Hand}_E &\sqsubseteq \mathrm{Hand}_S, \\
\mathrm{Hand}_P &\sqsubseteq \mathrm{Hand}_S \sqcap \exists \mathrm{part}.\mathrm{Hand}_E, \\
\mathrm{UpperLimb}_E &\sqsubseteq \mathrm{UpperLimb}_S, \\
\mathrm{UpperLimb}_P &\sqsubseteq \mathrm{UpperLimb}_S \sqcap \exists \mathrm{part}.\mathrm{UpperLimb}_E, \\
\mathrm{Finger}_S &\sqsubseteq \mathrm{Hand}_P, \\
\mathrm{Hand}_S &\sqsubseteq \mathrm{UpperLimb}_P.
\end{aligned}
$$

Figure 1: A fragment of $\mathcal{O}_{\mathrm{Amp}}$ [1, Figure 1], a consistent but erroneous ontology.

The ontology $\mathcal{O}_{\mathrm{Amp}}$ from Figure 1, part of the medical ontology SNOMED CT [2], is consistent, but it implies the following fact:

$$\mathrm{FingerAmputation} \sqsubseteq_{\mathcal{O}_{\mathrm{Amp}}} \mathrm{HandAmputation},$$

according to which every finger amputation is also a hand amputation. This does not cause an explicit inconsistency $\bot$ in $\mathcal{O}_{\mathrm{Amp}}$, but an **undesired consequence** in its application domain.

Not All DLs Can Express Inconsistencies

$\mathcal{EL}$, a lightweit DL characterized by efficient decision procedures for the *instance checking* and the *subsumption* problem [3], cannot express inconsistencies, because $\bot$ is not part of its syntax:

Definition $C \overset{\text{def}}{=} \top \mid C \sqcap C \mid \exists r.C$ is the grammar of $\mathcal{EL}$ concepts.

Consequently, a way to repair unwanted consequences in $\mathcal{EL}$ is also needed.

Contributions

The approach of [4], connecting different notions of **repair** from **prioritized KBs** to **abstract argumentation frameworks** (AAFs), is extended to allow the removal of an arbitrary assertion, not just an inconsistency. To this end, the notion of repair and conflict is generalized to arbitrary assertions. Given a KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ and a concept assertion α

- an **α-repair** is a $\subseteq$-maximal subset of $\mathcal{A}$ that does not $\mathcal{T}$-entail α;
- an **α-conflict** is a $\subseteq$-minimal subset of $\mathcal{A}$ that $\mathcal{T}$-entails α.

The two notions of priority relation and prioritized KB are accordingly modified, too:

- an **α-priority relation** for $\mathcal{K}$ and α is an acyclic relation $\succ_\alpha \subseteq \mathcal{A} \times \mathcal{A}$ such that if $\beta \succ_\alpha \gamma$, then an α-conflict $C \in \mathrm{Conf}_\alpha(\mathcal{K})$ exists such that $\{\beta, \gamma\} \subseteq C$.
- an **α-prioritized KB** is a pair $\mathcal{K}_{\succ_\alpha} = (\mathcal{K}, \succ_\alpha)$, where $\mathcal{K}$ is a KB and $\succ_\alpha$ an α-priority relation for $\mathcal{K}$ and α.

This allows generalizing [4, Theorem 35] from ordinary conflicts to α-conflicts:

Theorem 1 Given an α-prioritized $\mathcal{EL}$ KB $\mathcal{K}_{\succ_\alpha} = ((\mathcal{T}, \mathcal{A}), \succ_\alpha)$, for an assertion α not $\mathcal{T}$-entailed by any single assertion in $\mathcal{A}$, $S \subseteq \mathcal{A}$ is a Pareto-optimal α-repair of $\mathcal{K}_{\succ_\alpha}$ iff S is a stable extension of $F_{\mathcal{K},\succ_\alpha}$.

Proof idea Given the α-prioritized $\mathcal{EL}$ KB $\mathcal{K}_{\succ_\alpha} = ((\mathcal{T}, \mathcal{A}), \succ_\alpha)$, the prioritized $\mathcal{ELO}_\bot$ KB $\mathcal{K}'_{\succ_\alpha} \overset{\text{def}}{=} ((\mathcal{T}', \mathcal{A}), \succ_\alpha)$ is associated to it, where $\mathcal{T}' \overset{\text{def}}{=} \mathcal{T} \cup \{\{a\} \sqcap C \sqsubseteq \bot\}$. It can be argued that a set $S \subseteq \mathcal{A}$ is an α-conflict in $\mathcal{K}_{\succ_\alpha}$ iff it is a conflict in $\mathcal{K}'_{\succ_\alpha}$. By [4, Theorem 35], S is a conflict in $\mathcal{K}'_{\succ_\alpha}$ iff it is a stable extension in $F_{\mathcal{K}',\succ_\alpha}$. But $F_{\mathcal{K}',\succ_\alpha} = F_{\mathcal{K},\succ_\alpha}$, so S is a stable extension in $F_{\mathcal{K}',\succ_\alpha}$ iff it is a stable extension in $F_{\mathcal{K},\succ_\alpha}$. ∎

$$
\begin{tikzcd}
\mathcal{EL}\ \text{KB}\ \mathcal{K}_{\succ_\alpha} \arrow[r, "\mathcal{T} \longmapsto \mathcal{T} \cup \{\{a\} \sqcap C \sqsubseteq \bot\}"] \arrow[d, "\alpha\text{-conflict}"'] & \mathcal{ELO}_\bot\ \text{KB}\ \mathcal{K}'_{\succ_\alpha} \arrow[d, "\text{conflict}"] \\
\text{PSETAF}\ F_{\mathcal{K},\succ_\alpha} \arrow[r, phantom, "="] & \text{PSETAF}\ F_{\mathcal{K}',\succ_\alpha}
\end{tikzcd}
$$

Figure 2: Depiction of the proof of Theorem 1.

Beyond Classical Repairs

Classical repairs, subsets of a KB that do not entail an inconsistency or an unwanted consequence, are only one type of repairs to which AAFs can be associated. What seen here are only the first steps to connect AAFs with other forms of repairs, like **optimal repairs** [5] and **gentle repairs** [6]:

- **Optimal repair**: a **syntax-independent** form of repair that introduces no new consequences and retains as many assertions as possible;
- **Gentle repair**: a form of repair which modifies the axioms of a KB without removing them.

References

[1] F. Baader and B. Suntisrivaraporn, “Debugging SNOMED CT Using Axiom Pinpointing in the Description Logic $\mathcal{EL}$+,” in *KR-MED 2008*, pp. 1–7.

[2] “SNOMED International.” Accessed: Jul. 13, 2026. [Online]. Available: https://www.snomed.org/

[3] F. Baader, S. Brandt, and C. Lutz, “Pushing the $\mathcal{EL}$ Envelope,” in *IJCAI 2005*, pp. 364–369.

[4] M. Bienvenu and C. Bourgaux, “Querying and Repairing Inconsistent Prioritized Knowledge Bases: Complexity Analysis and Links with Abstract Argumentation,” in *KR 2020*, pp. 141–151.

[5] F. Baader, P. Koopmann, and F. Kriegel, “Optimal Repairs in the Description Logic $\mathcal{EL}$ Revisited,” in *JELIA 2023*, pp. 11–34.

[6] F. Baader, F. Kriegel, A. Nuradiansyah, and R. Peñaloza, “Making Repairs in Description Logics More Gentle,” in *KR 2018*, pp. 319–328.